NIPS Sesión 2

$\boxed{2}$ Explicit Euler
$$\begin{array}{c|c} 0 & 0 \\ \hline & 1 \end{array}$$
Order 1

Implicit Euler
$$\begin{array}{c|c} 1 & 1 \\ \hline & 1 \end{array}$$
Order 1

Implicit midpoint
$$\begin{array}{c|c} 1/2 & 1/2 \\ \hline & 1 \end{array}$$
Order 2

Implicit trapezoidal
$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1/2 & 1/2 \\ \hline & 1/2 & 1/2 \end{array}$$
Order 2

$$\begin{array}{c|cc} c_1 & a_{11} & a_{1s} \\ \vdots & & \\ c_s & a_{s1} & a_{ss} \\ \hline & b_1 & b_s \end{array}$$

Order 1 : $\displaystyle\sum_{i=1}^{s} b_i = 1$

Order 2 : $\displaystyle\sum_{i,j=1}^{s} b_i a_{ij} = 1/2$

Order 3 : $\begin{cases} \displaystyle\sum_{i,j,k=1}^{s} b_i a_{ij} a_{ik} = 1/3 \\ \displaystyle\sum_{i,j,k=1}^{s} b_i a_{ij} a_{jk} = 1/6 \end{cases}$

Runge method :
$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & 1/2 & 1/2 \end{array}$$
Order 2

$\boxed{3}$

$$\gamma(T)\cdot\phi(T) = 1 \qquad \forall T: |T| \le k$$

$$\begin{array}{c|ccc} & 0 & \cdots & 0 \\ & a_{21} & \ddots & \vdots \\ & a_{s1} & a_{ss-1} & 0 \\ \hline & & & \end{array}$$

$$|T| = s+1$$

$$\phi(T) = \sum_{m_1=2}^{s} \sum_{m_2<m_1} \cdots \sum_{m_{s-1}<m_s} b_{m_1} a_{m_1 m_2}$$

Check "Exercise 2_3" for the continuation

Alternative for exercise 3:

A RK method is of order p if for all sufficiently smooth f

$$\| y_1 - y(t_0+h) \| \sim O(h^{p+1})$$

Let's consider a particular problem

$$(1)\begin{cases} y'(t) = y(t) \\ y(0) = 1 \end{cases} \qquad (f(y) = y \quad f = id)$$

With exact solution : $y(t) = e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$

Let's apply an <u>explicit</u> RK method to (1)

$$\underline{a} = \begin{pmatrix} 0 & & 0 \\ a_{21} & \ddots & \\ \vdots & & \\ a_{s1} & \cdots a_{ss-1} & 0 \end{pmatrix}$$

$k_1 = f(y_0) = y_0 = 1$ (polynomial of order 0 in h)

$k_2 = f(y_0 + h a_{21} k_1) = 1 + h a_{21}$ (order 1 in h)

$$k_3 = f(y_0 + h(a_{31}k_1 + a_{32}k_2)) = \underbrace{1 + h(\underset{\in \mathbb{P}^0}{a_{31}k_1} + \underset{\in \mathbb{P}^1}{a_{32}k_2})}_{\in \mathbb{P}^2}$$

$k_s \in \mathbb{P}^{s-1}$ (i.e. a polynomial of order $s-1$ in h)

$y_1(h) = y_0 + h\sum_{i=1}^{s} b_i k_i$ is a polynomial of order s in h

$\| y(h) - y_1(h) \| = \| \sum_{k=0}^{\infty} \frac{1}{k!} t^k - y_1(h) \|$ at least is $O(h^{s+1})$

At most, an explicit RK method has order s

Its order is $p \leq s$